Logical Implication as an Object and Proficiency in Proof by Mathematical Induction

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Abstract

Proof by mathematical induction is known to be conceptually difficult for undergraduate students. We present a model that may simulate the impact of logical implication on students mastering proof by induction. We combine Piaget's action-object theory of mathematical development with a psychological model of working memory. We analyzed three sets of written assessments from two Introduction to Proofs classes: after students learned about logical implication; before and after instruction on proof by induction. We examine the relationship between proficiency with mathematical induction and treating logical implication as an object within these two classes.

Research Question

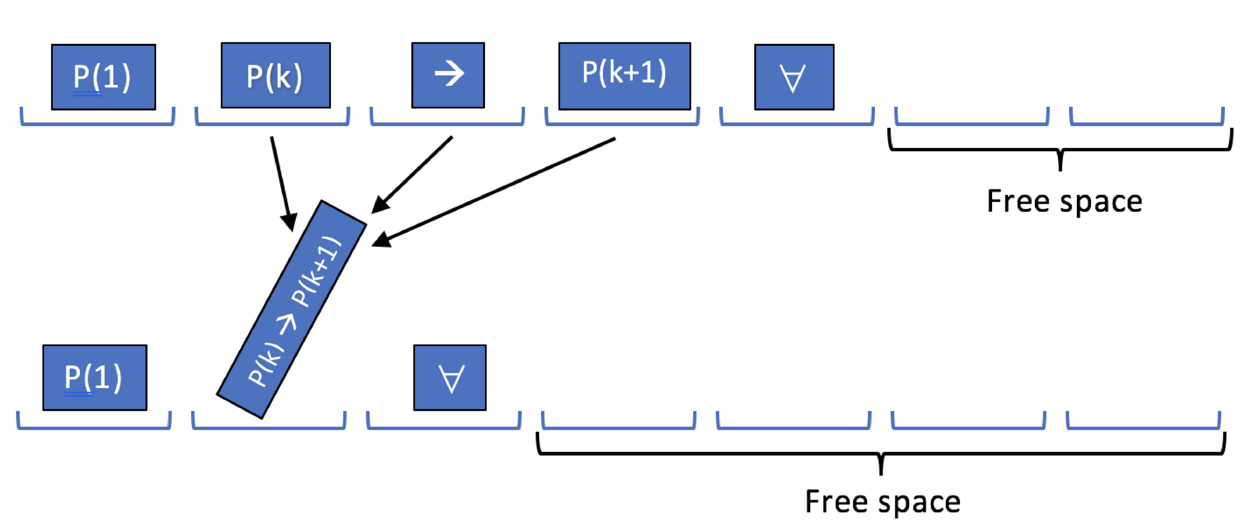
What is the impact of holding logical implication as a mathematical object on students' responses to formal instruction on proof by induction?

Theoretical Framework

We apply Piagetian (1970) *action-object theory* to inductive implication $P(k) \to P(k+1)$.

Students' mastering proof by mathematical induction heavily relies on their *encapsulation* of logical implication as a *mental object* (Dubinsky, 1986).

An encapsulation of the components of logical implication as a single mental object is parallel to what cognitive psychologists call "*chunking*" (Pascual-Leone, 1970). Chunking offloads cognitive demands on working memory.



Our framework is built on the conjecture that the combination of cognitive units together with their logical structure not only offloads working memory, but also facilitates the development of new mathematical understanding (Norton & Arnold, 2018).

Tasks

Assessment	Task
Logical Implication (LI)	Let P and Q be events that have some nonzero probability of occurring, and suppose that the following two implications are true: • If P and Q are mutually exclusive, their probabilities are not independent. • If the probabilities of P and Q are independent, the probability of P and Q is the product of the probability of P and the probability of Q (a) What can you conclude if P and Q are independent? (b) What can you conclude if the probability of P and Q is not the product of the probability of P and the probability of Q? (c) What can you conclude if P and Q are not mutually exclusive?
Pre-Math Induction (MI)	For each of the following parts, decide whether the given information is enough to conclude that the following claim is true. **<u>Claim:</u> P(n) is true for all $n \in \mathbb{Z}^+$** If the given information is not enough, offer a brief explanation on why: • P(1) is true and there is an integer $k \geq 1$ such that $P(k) \to P(k+1)$. • P(1) is true and for all integers $k \geq 1,\ P(k) \to P(k+1)$. • For all integers $k \geq 1, P(k) \to P(k+1)$. • P(1) is true and for all integers $k \geq 2,\ P(k) \to P(k+1)$.
Post-MI	Suppose that the following statement is true: "If the two sets each have "property X", then the union of the two sets also has "property X". Prove the following claim: **<u>Claim:</u>** Given a finite collection of sets that each have "property X", the union of all of the sets in the collection also has "property X".

Data Source

The participants are the students from two different classes of an Introduction to Proofs course, respectively taught by the second and the third authors. The course is a junior-level mathematics course designed to teach mathematics major students typical mathematical proof techniques.

Results: Chi-Squared Test

2nd Author $P = 0.017062$ $\chi^2 = 5.6902$		Post-MI Proficiency	
		Yes	**No**
LI Group	**Object**	7	4
	Action	1	8

3rd Author $P = 0.64716$ $\chi^2 = 0.20952$		Post-MI Proficiency	
		Yes	**No**
LI Group	**Object**	4	7
	Action	3	8

Discussion

There is a relationship between Post-MI proficiency and treating LI as an object among the 2nd author's students.

In contrast, there was no indication of such a relationship for the 3rd author's students.

This incompatibility could be explained by the fact that the 2nd author's class received formal instruction on quantification prior to instruction on induction, whereas the 3rd author's class did not.

References

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