Logical Implication as an Object and Proficiency in Proof by Mathematical Induction

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Proof by mathematical induction is known to be conceptually difficult for undergraduate students. We present a model that may simulate the impact of logical implication on students mastering proof by induction. We combine Piaget's action-object theory of mathematical development with a psychological model of working memory and Harel and Sowder's proof schemes. We analyzed three sets of written assessments from two Introduction to Proofs classes: after students learned about logical implication; before and after instruction on proof by induction. We examine the relationship between proficiency with mathematical induction and treating logical implication as an object within these two classes.

Keywords: Logical Implication, Mathematical Induction, Proof, Action-Object Theory.

Proof by mathematical induction is known to be conceptually difficult for undergraduate students. This method is used to prove that the statement P(n) holds for any natural number n. To prove P(n) by mathematical induction, one must check two assumptions: (a) the validity of P(1) (the base case), and (b) if the statement P(k) is true for some natural number k, than it is also true for P(k+1) (inductive implication). The implication $P(k)\rightarrow P(k+1)$ may be considered as either an operator that transforms P(k) into P(k+1) or as an invariant relationship between P(k) and P(k+1) (Norton & Arnold, 2017).

Our study is guided by the following research question: "What is the impact of holding logical implication as a mathematical object on students' responses to formal instruction on proof by induction?". We describe a model that may simulate the impact of logical implication on students mastering proof by mathematical induction. We draw on Piaget's (1970) action-object theory of mathematical development. More specifically, we elaborate Dubinsky's (1991) hypothesis that treating implication as an object is crucial for promoting students' understanding proof by induction. Additionally, our framework incorporates psychological model of working memory, and Harel and Sowder's (2007) proof schemes.

Participants were students from two classes of an Introduction to Proofs course, taught separately by the second and the third authors. The students completed three written assessments: 1) after the students learned logical implication, 2) just before instruction on proof by induction, and 3) after instruction on proof by induction. We used a chi-squared test to assess the relationship between post-mathematical induction (Post-MI) proficiency and treating logical implication (LI) as an object in the 2nd author's students; however, there was no indication of a relationship in the 3rd author's students (see Table 1). During the poster session, we will discuss reasons for this difference. For example, the 2nd author's class received formal instruction on quantification prior to instruction on induction, whereas the 3rd author's class did not.

Table 1. Chi-Squared Test.

2nd Author $p = 0.03995$		Post-MI Proficiency	
		Yes	No
LI Group	Object	7	4
	Action	1	6

3rd Author $p = 1.16883$		Post-MI Proficiency	
		Yes	No
LI Group	Object	6	5
	Action	2	5

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